ORBIT DETERMINATION OF CHANDRAYAAN-1 USING LUNAR LASER RANGING INSTRUMENT (LLRI) MEASUREMENTS

N.V.Vighnesam [1] ,Anatta sonney [2], and N.S.Gopinath[3]

[1] *Head, Orbit Dynamics Division, ISRO Satellite Centre, Bangalore, India, Email: vignes@isac.gov.in*
[2] *Engineer, Orbit Dynamics Division, ISRO Satellite Centre, Bangalore, India.*
[3] *Group Director, Flight Dynamics Group, ISRO Satellite Centre, Bangalore, India*

ABSTRACT

India's first Moon mission Chandrayaan-1 carrying eleven scientific instruments was launched on 22nd October 2008. Lunar Laser Ranging Instrument (LLRI) is one of the scientific instruments carried by Chandrayaan-1. It is an instrument aimed to enhance the study of the Moon's surface. LLRI instrument was commissioned on 16th November 2008. The ISRO's mission operational orbit determination (OD) software was suitably updated to estimate the Chandrayaan-1 orbit with LLRI measurements. This paper describes the method involved in OD process and orbit solutions with simulated as well as live LLRI data of Chandrayaan-1. This work resulted in obtaining satisfactory results with LLRI measurements.

1. INTRODUCTION

India's first Moon mission Chandrayaan-1 carrying eleven scientific instruments for the purpose of expanding scientific knowledge about the Moon was launched on 22nd October 2008 from Satish Dhawan Space Centre, Sriharikota, India by India's Polar Satellite Launch Vehicle PSLV-C11. Spacecraft was injected into transfer orbit of (254.4 X 22932.7) km with inclination of 17.9 deg at 2008-10-22-01-10-19-081 UT. The main objective of the mission is a simultaneous chemical, mineralogical and photo geologic mapping of the whole Moon with high spatial resolution using high resolution state of the art sensors.

The spacecraft was put into Moon's polar, circular orbit of about (100 X100) km by carrying out five Earth bound maneuvers (EBNs), one trajectory correction (TCM) and four Lunar bound maneuvers (LBNs) on 12th November 2008. Lunar Laser Ranging Instrument (LLRI) is one of the eleven scientific instruments carried by Chandrayaan-1. It is a pulsed laser-ranging instruments aimed at enhanced study of the Moon's surface. LLRI measurements can be obtained from both the dark and sunlit portion of the Moon, thereby significantly increasing the useful observational coverage. LLRI instrument for Chandrayaan-1 capable of making lunar topography measurements with a resolution of less than 5 meters was successfully turned on 16th November 2008 at 03:47:088 UT. The purpose of this paper is to study the feasibility of Chandrayaan-1 orbit determination using LLRI measurements.

ISRO's mission operational orbit determination (OD) software [1] was suitably updated to determine the orbit with LLRI measurements and demonstrated using simulated as well as actual LLRI measurements obtained during initial phase of the mission.

The main features of orbit determination are Trajectory Generation, Observation Modeling and Estimation. The main difference between OD with tracking data and with LLRI data is observation modeling. This paper describes the method involved in observation modeling in detail and OD results with simulated as well as actual/live LLRI data. The comparison study was made between OD results

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using tracking data (range and Doppler data) and using LLRI data. The LLRI based OD results will substitute in case of either non-availability of tracking data or non-availability of converged state with tracking data. The work on updating the Chandrayaan-1 operational orbit determination program using tracking data for LLRI measurements resulted in very satisfactory OD results.

2. ORBIT DETERMINATION SYSTEM

A precise knowledge of orbital parameters is a prerequisite for the determination of the position of satellite at any given time. Mathematically orbit is specified by a set of six orbital parameters which completely describes the motion of the satellite within the specified accuracy over a period of time. Determination of the satellite orbit means the refinement of the initial orbit parameters. The problem of OD consists of comparing measurements taken on a satellite trajectory to a model representing that trajectory. The model is generally represented by a system of differential equations whose constants of integration define the satellite trajectory as a function of time. Thus, given a priori estimate of these constants of integration (state parameters), the problem is to update or correct the initial (a priori) state parameters as a function of measurements. The orbit determination system consists of tracking data system and data handling, dynamic models, computational techniques, which include trajectory generation and estimation technique.

The main computation process of the orbit determination system is trajectory generation, observation modeling and estimation. Trajectory generation is performed through numerical integration of differential equations of motion of satellite. The force model for artificial satellites normally includes gravitation attraction of all planets, aerodynamic drag, luni-solar gravitation and solar radiation pressure. Cowell's method is used for trajectory generation. Weighted least squares technique and iterative differential correction process is used to obtain the refined state. The method of integration of equations of motion and estimation are described in the following sections.

2.1 Main features of Orbit Determination System

The main features of orbit determination system include measurement data processing, trajectory generation, observation modeling and orbit estimation. Generally observational data is tracking data. The present study of obtaining orbit determination is based of LLRI measurements.

2.1.1 Method of Integration for Trajectory Generation

The coupled nonlinear second order differential equations of motion are integrated numerically through Cowell's method. The chief advantage of special perturbation technique is its high accuracy. The perturbative acceleration acting on the satellite is modeled. Numerical integration methods are customarily classified into two parts namely single step and multi step methods. Multi step methods normally need a starter. The single step methods that are considered are those of Runge Kutta or RK-Gill etc. Multi step methods are "Adams" or its variants. Exclusively for solving second order equations, double integration method is the recommended procedure. These methods are stable. Gauss-Jackson-Merson's (GJM) 8^{th} order method is employed here [2].
The basic equations to solve the equation of motion

$$\ddot{x} = f(x, \dot{x}, t) \tag{1}$$

are

$$x_{i+1} = x(t_i + h) = h^2(\nabla^{-2}\ddot{x}_i + \frac{1}{12}\ddot{x}_i + \frac{1}{12}\nabla\ddot{x}_i + \frac{19}{240}\nabla^2\ddot{x}_i + \frac{1}{40}\nabla^3\ddot{x}_i + \frac{863}{12096}\nabla^4\ddot{x}_i + \ldots)$$
$$\dot{x}_{i+1} = \dot{x}(t_i + h) = h(\nabla^{-1}\ddot{x} + \frac{1}{2}\ddot{x}_i + \frac{5}{12}\nabla\ddot{x}_i + \frac{3}{8}\nabla^2\ddot{x}_i + \frac{251}{720}\nabla^3\ddot{x}_i + \frac{95}{288}\nabla^4\ddot{x}_i + \frac{19087}{60480}\nabla^5\ddot{x}_i + \ldots) \quad (2)$$

in standard backward difference notation.

The Gauss-Jackson corrector formulae are

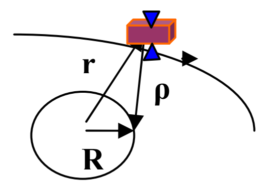
$$x_{i+1} = h^2(\nabla^{-2}\ddot{x}_i + \frac{1}{12}\ddot{x}_{i+1} - \frac{1}{240}\nabla^2\ddot{x}_{i+1} - \frac{1}{240}\nabla^3\ddot{x}_{i+1} - \frac{221}{60480}\nabla^4\ddot{x}_{i+1} - \frac{19}{6048}\nabla^5\ddot{x}_{i+1} - \frac{9829}{60480}\nabla^6\ddot{x}_{i+1} + \ldots\ldots)$$

$$\dot{x}_{i+1} = h(\nabla^{-1}\ddot{x}_i + \frac{1}{2}\ddot{x}_{i+1} - \frac{1}{12}\nabla\ddot{x}_{i+1} - \frac{1}{24}\nabla^2\ddot{x}_{i+1} - \frac{19}{720}\nabla^3\ddot{x}_{i+1} - \frac{3}{160}\nabla^4\ddot{x}_{i+1} - \frac{275}{24192}\nabla^6\ddot{x}_{i+1)} + \ldots.) \quad (3)$$

2.1.2 Observation Modeling

For a given LLRI measurement, the theoretical/computed measurement ρ_c is obtained using satellite definitive ephemeris and spacecraft quaternions (q_s) along with instrument misalignment angles.

The following steps are followed to obtain theoretical measurement:



The observed LLRI measurement ρ_o, the scalar value is converted into inertial vector $\boldsymbol{\rho}_o$, using direction cosine (DC) matrix obtained from spacecraft quaternions and instrument misalignment matrix.

$\boldsymbol{\rho}_o = \rho_o$ [DC matrix obtained from q_s] [instrument misalignment matrix]

The vector **R**, from Moon's centre to Moon's surface is given as $\mathbf{R} = \mathbf{r} + \boldsymbol{\rho}_o$, where **r** is the spacecraft position vector from definitive ephemeris (inertial EME-J2000). **R** is converted from selenocentric to selenographic to compute longitude λ and latitude φ of LLRI measurement on the lunar surface.

The Moon's radius R at a given longitude and latitude is calculated using the following relation

$$R = \sum\sum P_{lm}(\sin\varphi)(C_{lm}\cos(m\lambda) + S_{lm}\sin(m\lambda)) \quad (4)$$

Where, P_{lm} is the Lagrangian coefficient of order l, m;
φ and λ are the latitude and longitude;
C_{lm}, S_{lm} are topographic coefficients

The computed R is converted into vector **R** using λ and φ and converted into inertial (EME J2000) $\mathbf{R_i}$.
Computed LLRI measurement $\boldsymbol{\rho}_c$ using $\mathbf{R_i}$ and **r** is $\boldsymbol{\rho}_c = \mathbf{R_i} - \mathbf{r}$.

2.1.3 Estimation

Optimal estimate of satellite state is computed from a given vector of tracking measurements which depends on the instantaneous position and velocity of spacecraft. The estimated state is valid over the period during which measurement data is collected.

The general procedure for all definitive orbit computations is to set up some dynamical model of the orbit and use the observations to improve the orbit parameters of the model by the process of differential correction. Weighted Least Squares estimation process is applied. Observations are considered for some pre-selected time period and by differential correction of parameters of the model, the sum of the squares of the residual is minimized. The basic idea of least-squares estimation as applied to orbit determination is to find trajectory and the model parameters for which the square of the difference between the modeled observations and the actual measurements becomes as small as possible. In this estimation process of "Weighted Least Squares" it is necessary to compute partial derivatives of observations with respect to model parameters. Brief description of method of estimation adopted in this present study is as follows.

Given *m* observations and an orbit that is to be corrected using these observations consisting of *n* parameters, $P_j, j = 1,...n$, and if M_o is the observed quantity, M_c is the corresponding computed quantity from the mathematical model, then each of the observed quantity can be considered as a function of state parameters, the station coordinates, time and if necessary of other relevant parameters affecting the motion of the satellite. In this context, only the dependence of orbital parameters is of importance.

Thus,

$$M_c = M_c\{P\} \tag{5}$$

By comparing the computed quantities with *m* observed quantities, the residuals are given by,

$$\Delta M_i = M_{oi} - M_{ci}\{P_j\}, i = 1,2,....m \tag{6}$$

The object is to obtain expressions in terms of known quantities. The residuals after the first iteration become:

$$\Delta M^{'} = M_{oi} - M_{ci}\{P_j^{'}\}, i = 1,2,...m \tag{7}$$

where,

$$P_j^{\prime} = P_j + \Delta P_j \ .$$

Expanding by Taylor series and linearizing, the conditional equations become:

$$\Delta M^{'}{}_i = \Delta M_i - \sum_{j=1}^{n} \frac{\partial M}{\partial P_j}, j = 1,2,..m \tag{8}$$

By multiplying weighting factors, for all *m* observed quantities the equations of condition can be written in matrix notation as:

$$W_1 R' = W_1 R - W_1 A \Delta P \tag{9}$$

where,

W_1 is the weighting matrix of mean measurement error (total expected error for the each measurement type due to both random noise and systematic errors) which is derived from the covariance matrix of observations by taking square root of the diagonal elements. R' is the column matrix of residuals after differential correction, R is the column matrix of residuals before differential correction, A is the matrix of partial derivatives.

If everything is ideal, R' should contain all zeros after just one iteration. The solution for ΔP would have been trivial if there were same observed quantities as that of parameters to be refined. In practice, 'm' is much greater than 'n' and the second order terms are not negligible, so that one iteration does not suffice. Therefore, weighted least square technique is adopted. The condition of the least square approach is:

$$[W_1 R']^T [W_1 R'] \quad \text{is minimum} \tag{10}$$

that leads to the normal equations:

$$A^T W_1^{\ T} (W_1 A \Delta P - W_1 R) = 0 \tag{11}$$

Letting
$W = W_1^{\ T} W_1$,

the equation yields:

$$\Delta P = (A^T W A)^{-1} A^T W R \tag{12}$$

ΔP is the correction to P for the current iteration. This process is to be carried out for a number of iterations till the solution converges.

3. MEASUREMENTS USED FOR ORBIT DETERMINATION

Orbit determination using LLRI measurements is demonstrated with Chandrayaan-1 simulated as well as actual/live data. About two days of LLRI measurements were simulated using Chandrayaan-1 state parameters given in Table 1.

Table 1 Measurements Simulation

Epoch : 2008-12-18-15-48-54-816
Start of measurement : 2008-12-18-15-49-54-816
End of measurement : 2008-12-20-15-18-54-816

State parameter (km, km/s, EME-J2000)	
X.Y,Z	68.485, -278.061. 1828.229
XD,YD,ZD	-0.111485 , -1.603584 , -0.230156

Live LLRI data obtained on 16th November 2008 (1st day of commissioning LLRI instrument) and 1st week of February 2009 was used to demonstrate OD. Orbit determination results using 16th November 2008 LLRI data are shown in Table 3. These OD results are compared with orbit solutions obtained using tracking data. Table 4 gives summary of OD results comparison using 1st week of February 2009 LLRI data.

4. OD RESULTS AND COMPARISON

Orbit determinations were carried out with simulated and actual Chandrayaan-1 LLRI data and comparison study was carried out with OD results obtained using tracking data (Range and accumulated Doppler data)

Table 2 Orbit determination summary using simulated data

EPOCH : 2008 12 18 15 48 54 816

TRACKING DATA FROM : 2008 12 18 15 49 54 816
TO : 2008 12 20 15 18 54 816
TRACKING DATA DURATION : 47.48 hours
No. of LLRI Measurements : 2850

Definitive State Parameters (EME-J2000)

Converged Versus Actual State (km, km/s)

Actual :	68.485	-278.061	1828.229	-0.111485	-1.603584	-0.230156
Realized:	68.471	-278.111	1828.222	-0.111467	-1.603579	-0.230201
Difference (Actual – Realized) :	0.014	0.05	0.007	-0.000018	-0.000005	0.000045

Table 3 Orbit determination summary using actual LLRI Data

EPOCH : 2008 11 16 00 02 00 000
TRACKING DATA FROM : 2008 11 16 04 28 48 401
TO : 2008 11 16 17 23 39 442
TRACKING DATA DURATION : 12.91 hours

Definitive Orbit (EME J2000)

Orbit Comparison

	a (met)	e	i (deg)	ω (deg)	Ω (deg)	M (deg)
OD with tracking data. :	1842015.732578	0.005425	93.106987	260.379444	86.307290	304.758661
OD with LLRI data :	1841976.110290	0.005396	93.342136	261.742166	86.517621	303.472880
Difference :	39.622288	0.000029	0.235149	1.362722	0.210331	1.285781

Table 4 Orbit comparison (LLRI OD Vs. Tracking data OD)

Epoch	Orbit with Trk.data			Orbit with LLRI data			Difference		
	'a'(km)	'e'	'i'(deg)	'a' (km)	'e'	'i'(deg)	'a'(km)	'e'	'i'(deg)
2009 02 01	1839.94328	0.004194	92.8556	1839.95064	0.004225	92.9648	0.00736	0.31e-4	0.1092
2009 02 02	1840.65106	0.003560	92.8873	1840.69025	0.003644	92.8527	0.03919	0.85e-4	0.0346
2009 02 03	1839.43615	0.002965	93.1628	1839.42036	0.003023	93.2679	0.01579	0.58e-4	0.1051
2009 02 04	1839.94554	0.002327	93.4794	1839.99136	0.002318	93.4702	0.04582	0.90e-5	0.0092
2009 02 05	1840.33579	0.003244	93.5257	1840.34211	0.003298	93.4699	0.00633	0.55e-4	0.0558
2009 02 06	1839.28389	0.003485	93.4894	1839.30033	0.003523	93.2542	0.01644	0.38e-4	0.2352
2009 02 07	1840.41730	0.005942	93.6716	1840.42967	0.006139	93.6867	0.01237	0.20e-3	0.0151
2009 02 08	1839.46514	0.008093	93.6718	1839.44635	0.008272	93.7944	0.01878	0.18e-3	0.1226

5. CONCLUSION

The LLRI based OD results can substitute in case of either non-availability of tracking data or non-availability of converged state with tracking data. The work on updating the Chandrayaan-1 operational orbit determination program using tracking data (range and Doppler data) for LLRI measurements resulted in very satisfactory OD results.

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